

2020
MATHEMATICS
[HONOURS]
Paper : VIII

Full Marks : 50

Time : 2 Hours

The figures in the right-hand margin indicate marks.
Symbols and Notations have their usual meaning.

GROUP-A
(Numerical Analysis)
(Marks : 30)

1. Answer any **two** questions: 1×2=2
- Prove that $E = \Delta + 1$
 - State the condition of convergence of Newton-Raphson method.
 - What do you mean by degree of precision of a quadrature formula?
 - State the composite form of Simpson's $\frac{1}{3}$ rd rule for numerical integration.
2. Answer any **three** questions: 2×3=6
- Show that $\Delta \log f(x) = \log \left\{ 1 + \frac{\Delta f(x)}{f(x)} \right\}$

[Turn over]

- State under what conditions the 4th order Runge-Kutta method reduces to Simpson's $\frac{1}{3}$ rd rule.
 - Find the value of $y(x=1.1)$ for the problem $\frac{dy}{dx} = 1 + xy, y(1) = 1$ by using Euler's method.
 - If $f(x) = \frac{1}{x^2}$, whose arguments are a, b, c then prove that $f[a, b, c] = \frac{ab + bc + ca}{a^2b^2c^2}$.
 - Define polynomial interpolation. Comment on its uniqueness.
3. Answer any **two** questions: 6×2=12
- Show that the divided differences of order n are symmetric functions of their arguments.
 - Show that the n^{th} difference of a real polynomial $p(x)$ of degree n is constant and $(n+1)^{\text{th}}$ difference vanishes. 3+3
 - Explain the bisection method for finding a real root of an equation $f(x)=0$ and show that the method is unconditionally and surely convergent to the root, if it exists. 6
 - If $K_r^{(n)}, r = 0, 1, 2, \dots, n$ be the Cotes' numbers with reference to Newton-Cotes quadrature

formula, then show that $K_r^{(n)} = K_{n-r}^{(n)}$, $r = 0, 1, 2, \dots, n$ and comment on the significance of larger values of n .

- d) Explain the basic concepts used in the Gauss elimination approach for solving a linear system of n equations. Show that the number of multiplications and subtractions grows proportional to $\frac{n^3}{3}$ and the number of divisions proportional to $\frac{n^2}{2}$ from the computational effort involved.

- e) Solve the equation $\frac{dy}{dx} = \frac{x^2}{1+y^2}$ with the initial condition $y(0)=0$ by Picard's method to obtain y for $x=0.15, 0.25$ and 0.5 correct to 3 decimal places.

4. Answer any **one** question: $10 \times 1 = 10$

- a) i) Establish standard Runge-Kutta method of second order for numerical solution of the following initial value problem

$$\frac{dy}{dx} = f(x, y) \text{ with } y(x_0) = y_0. \quad 5$$

- ii) Use Runge-Kutta method to find an approximate value of y , when $x = 0.1$,

given that $\frac{dy}{dx} = x + y$, $y = 1$ when $x = 0$ with $h = 0.1$. 5

- b) i) Establish Lagrange's interpolation formula. If $x_0, x_1, x_2, \dots, x_n$ be the interpolating points and $\omega_i(x)$, $i = 0, 1, 2, \dots, n$ be the Lagrangian functions, then

$$\text{show that } \sum_{i=0}^n \omega_i(x) = 1. \quad 6$$

- ii) Show that $\Delta \binom{n}{x+1} = \binom{n}{x}$, where the forward difference operator Δ operates on n and hence show that

$$\sum_{n=1}^N \binom{n}{i} = \binom{N+1}{i+1} - \binom{1}{i+1}. \quad 4$$

GROUP-B
(Fundamentals of Computer Science and
Computer Programming)
(Marks : 20)

5. Answer any **two** questions: 2×2=4

- a) What do you mean by Machine language and Assembly language?
- b) Obtain the binary equivalents of $(13.5)_{10}$ and $(8.6)_{10}$ and compute their sum using binary arithmetic.
- c) What are keywords in C? Can keywords be used as an identifier?
- d) Express the algebraic expression in its

equivalent C expression $a = \frac{b^2 \sqrt{1+x^2} \log x}{\cos(x-b)}$.

6. Answer any **one** question: 6×1=6

- a) i) What is an array? How an array variable is different from an ordinary variable? 2
- ii) Write a C program to find the sum of two matrices of same order. 4
- b) i) What is CPU? What do you mean by a software? 2
- ii) Use 2's complement method to compute the difference $(1001.011)_2 - (111.111)_2$. 2

iii) Illustrate the difference between compiler and interpreter. 2

7. Answer any **one** question: 10×1=10

- a) i) Write short notes on 'switch' statement and 'break' statement. 5
- ii) Write a C program to count and display numbers between 1 to 100 not divisible by 2, 3 and 5. 5
- b) i) What is a loop? Why it is necessary in a C program? 2
- ii) How do you choose between 'while' and 'for' loop? 2
- iii) Write a C program to check if the entered year is a leap year or not. 6