2020
MATHEMATICS
[HONOURS]
Paper : VII

Full Marks : 100 Time : 4 Hours

The figures in the right-hand margin indicate marks.
Symbols and notations have their usual meanings.

GROUP–A

1. Answer any **five** questions: 1×5=5

a) Define a mixed tensor of rank 2.

b) If the relation $b_j^i v_i = 0$ holds for any arbitrary covariant vector v_i show that $b_j^i = 0$.

c) When a function f is called analytic at a point Z_0?

d) For two vectors $\vec{a}$ and $\vec{b}$ define $\vec{a} \times \vec{b}$.

e) Write axioms of probability.

f) State law of Large numbers.

g) Define sampling distribution of a statistics.

h) Define a solenoidal field.

[Turn over]

GROUP–B

2. Answer any **ten** questions: 2×10=20

a) Given an example with justification that two events are independent but not mutually exclusive.

b) If Δij is a skew-symmetric tensor, show that $\left(\delta_i^j \delta_i^k + \delta_i^j \delta_j^k\right)\Delta ij = 0$.

c) Show that $\nabla^2\left(\frac{1}{r}\right) = 0$ where $r = \sqrt{x^2 + y^2 + z^2}$ and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$.

d) Prove that $\vec{F} = \left(y^2 \cos x + z^3\right)\hat{i} + \left(2y \sin x - 4\right)\hat{j} + \left(2xz^2 + 2\right)\hat{k}$ is a conservative force field.

e) If $\vec{r} = a\cos t\,\hat{i} + a\sin t\hat{j} + bt\hat{k}$ find $\left|\frac{d^2\vec{r}}{dt^2}\right|$.

f) Find the fixed points of the bilenear transformation $\omega = \left((2+i)z - 2\right)/z + i$.

g) If $x = \frac{1}{2}\log\left(x^2 + y^2\right)$ is harmonic then find its harmonic conjugate.

h) Verify whether $f(z) = \log(z)$ is analytic at z=0.

i) Show that S_j^i is a mixed tensor of rank 2.

j) Six coins are tossed 6400 times, using Poisson distribution. Find the probability of getting six heads m times.

k) If $P(A+B)=\frac{5}{6}$, $P(AB)=\frac{1}{3}$ and $P(\overline{B})=\frac{1}{2}$ then prove that A and B are independent.

l) State central limit theorem.

GROUP–C

3. Answer any **five** questions: 6×5=30

a) If $b^{ij}u_i u_j$ is an invariant for arbitrary convariant vector u_i then show that $b^{ij}+b^{ii}$ is a contravariant tensor of rank 2.

b) Evaluate $\iint_S \vec{A}.\vec{n}\,ds$, where $\vec{A}=18z\hat{i}-12\hat{j}+3y\hat{k}$ and S is the part of the plane $2x+3y+6z=12$ which is located in the first octant.

c) State Green's theorem. Verify Green's theorem in plane $\oint_c (x^2+y^3)dx+(x^3+y^2)dy$ where C is the boundary of the pentagon with vertices (0, 0), (1, 0), (2, 1) (1, 2) and (0, 1)

d) Show that all odd ordered central moment of normal distribution is 0.

e) If the probability distribution of a discrete random variable X is given by $P(X=x)=ke^{-t}(1-e^{-t})^{x-1}$, $x=1,2,....\infty$, find the value of K and also the mean and variance of X.

f) If (X, Y) is a two-dimensional Random variable uniformly distributed over the triangular region R bounded by y = 0, x = 3 and $y=\frac{4}{3}x$. Find $f_x(x), f_y(y), E(x), Var(x)$, $E(Y)$ $Var(Y)$ and ρ_{xy}.

g) Show that the function $f(z)=e^{z^4}\ (z\neq 0)$ and $f(0)=0$ is not analytic at z = 0 although Cauchy-Riemann equations are satisfied at the point.

h) Determine the bilinear transformation which transforms the circle $|z|<\rho$ on to the circle $|\omega|<\rho'$.

GROUP–D

4. Answer any **three** questions: 15×3=45

a) i) Show that a second order covarient tensor can be expressed as a sum of a symmetric and a skew-symmetric tensor.

ii) Verify Stoke's theorem for

$\vec{F}=(2x+y)\hat{i}+yz^2\hat{j}+y^2z\hat{k}$ where S is the upper half of the sphere $x^2+y^2+z^2=1$ and C is the boundary.

iii) Verify the divergence theorem for $\vec{A}=4x\hat{i}-2y^2\hat{j}+z^2\hat{k}$ taken over the region bounded by $x^2+y^2=4$, $z=0$ and $z=3$. 5+5+5=15

b) i) Find a bilinear transformation which maps the circle $|\omega|\le 1$ into the circle $|z-1|<1$ and maps $\omega=0$, $\omega=1$ respectively into $z=\frac{1}{2}, z=0$. Show also that the transformation is uniquely determined.

ii) State and prove Cauchy-Reimann conditions of a function to be analytic in polar co-ordinate system.

iii) Find all circles which are orthogonal to $|z|=1$ and $|z-1|=4$. 5+5+5=15

c) i) If X is a Random variable with a continuous distribution function F(x), prove that Y=F(x) has a uniform distribution in (0, 1). Further if

$$f(x)=\begin{cases}\frac{1}{2}(x-1) & 1\le x\le 3\\ 0 & \text{otherwise}\end{cases}$$

find the range of Y corresponding to the range $1.1\le x\le 2.9$.

ii) A Random variable x(>0) possesses the property that P(X>x+h) = P(X>x). P(X>h) for all h > 0. Prove that X follows exponential distribution with density function $F(x)=\lambda e^{-\lambda x}$ where $\lambda=F^1(0)>0$, $F(x)$ is the distribution function of X.

iii) Find the mean and standard deviation of the distribution whose moment generating function is $(0.4e^t + 0.6)$ 5+5+5=15

d) i) Applying central limit theorem to a sequence of random variables with Poisson distribution, prove that $\lim_{n-\infty} e^{-n}\sum_{r=0}^{n}\frac{n^r}{r^1}=\frac{1}{2}$.

ii) The random variables X, Y are normally correlated with correlation coefficient ρ. Prove that $\frac{X}{\sigma x}+\frac{Y}{\sigma y}$ and $\frac{X}{\sigma x}-\frac{Y}{\sigma y}$ independent normal variates with variances $2(1+\rho)$ and $2(1-\rho)$ respectively.

iii) Assume that $A^i B_i$ is invariant for all contravariant vector A^i. Then show that B_i is covariant vector. 5+5+5=15

e) i) Define confidence interval. Find the confidence interval for population mean m for normal (m, σ) population where σ is known.

ii) Two independent samples of sizes 8 and 7 contained the following values:

Sample 1 :	19	17	15	21	16	18	16	14
Sample 2 :	15	14	15	19	15	18	16	

Is the difference between the sample means significant? Test it at 5% level of significance

Given $t_{0.05}(\gamma = 13) = 2.16$.

iii) Find Maximum Likelyhood estimate of μ and σ^2 for $N(\mu, \sigma^2)$ population.

5+5+5=15

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