2020
MATHEMATICS
[GENERAL]
Paper : IV

Full Marks : 100 Time : 3 Hours

The figures in the right-hand margin indicate marks.
Symbols and Notations have their usual meaning.

1. Answer any **six** questions: 1×6=6

a) Prove that $\Delta \bullet \nabla = \Delta - \nabla$.

b) What is interpolation?

c) When are Newton's forward and backward interpolation formulae used?

d) Subtract $(101011)_2$ from $(110001)_2$.

e) Using normalized floating point representation, add 0.4546E5 and 0.5433E7.

f) Write the laws of static friction.

g) Define amplitude and frequency of a Simple Harmonic Motion.

h) State the Kepler's laws of planetary motion.

i) What are the characteristics of a central orbit?

2. Answer any **eleven** questions: 2×11=22

a) Find the relative and percentage errors in an approximate representation of $\pi = 3.14159$ by $\frac{22}{7}$.

b) Evaluate $\left(\frac{\Delta^2}{E}\right)x^3$, spacing being one.

c) Write the Lagrange's interpolating polynomial for the three points (x_0, y_0), (x_1, y_1) and (x_2, y_2).

d) Show that the third differences of a quadratic function are zero.

e) Give the graphical representation of the method of false position.

f) What is the degree of the approximating polynomial corresponding to Trapezoidal rule and Simpson's $\frac{1}{3}$rd rule?

g) Derive $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$ for determining the square root of a > 0, using Newton-Raphson method.

h) What is binary number? Why do we need such numbers?

i) Write the decimal equivalence of the binary number $(11011.0011)_2$.

j) Differentiate between source program and object program.

k) What are the rules for naming a FORTRAN variable? What are those variables?

l) Show that the rate of change of K. E. of a particle is equal to its power.

m) The law of motion in a straight line being $x = A\cos(nt+k)$, where *x* is the position of the particle at any time *t*, *A*, *n* and *k* are constants; show that the acceleration varies as the distance.

n) The velocity *v* of a particle amoving along the x-axis is given by the relation $v^2 = n^2(8bx - x^2 - 12b^2)$, where *n* and *b* are constants. Prove that the motion is Simple Harmonic.

o) Write the magnitude and direction of radial and transverse components of acceleration.

p) A particle describes the curve $p^2 = ar$ under a force *F* which is directed towards the pole. Find the law of force.

3. Answer any **seven** questions: 6×7=42

a) What is meant by divided difference? Prove that the divided differences are symmetrical in all their arguments.

b) Explain the method of fixed point iteration for numerical solution of the equation of the form $x = \phi(x)$. Derive the condition of convergence.

c) Derive the formula of Newton-Raphson method for computing a simple real root of an equation $f(x) = 0$.

d) Draw a flowchart to find the roots of the equation $ax^2 + bx + c = 0$.

e) Write an algorithm to check if the three given points $(a_1, b_1), (a_2, b_2)$ and (a_3, b_3) are collinear.

f) Write a program in FORTRAN to find the product of two matrices of order $m \times n$ and $n \times l$.

g) Find the C.G. of the arc of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$ lying in the first quadrant.

h) Write down equations of motion of a particle moving in a central orbit under a central force

P and deduce the differential equation in the form $\frac{h^2}{p^3}\frac{dp}{dr} = P$; the symbols having usual meaning.

i) A body moving in a straight line OAB with Simple Harmonic Motion has zero velocity at the points A and B whose distances from O are *a* and *b* respectively and has velocity *V* when half way between them. Show that the complete period is $\frac{\pi(b-a)}{V}$.

j) A particle is placed on the outside of a rough sphere whose coefficient of friction is μ. Show that it will be on thepoint of motion when the radius from the centre makes an angle $\tan^{-1}\mu$ with the vertical.

k) A particle moves with a, central acceleration $\mu \div (distance)^3$. Find the path.

4. Answer any **three** questions: 10×3=30

a) i) Use Newton's forward interpolation formula to establish the formula

$$\left(\frac{d^2y}{dx^2}\right)_{x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 + \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \frac{137}{180}\Delta^6 y_0 + \ldots\right]$$

6

ii) Write an algorithm to calculate the area of an equilateral triangle. 4

b) i) Explain Bisection method to solve an equation of the form *f(x)* = 0. 6

ii) Write a FORTRAN program to find the maximum among three numbers. 4

c) i) Three forces act along the straight lines x =0, y–z = a; y = 0, z–x =a; z = 0, x–y = a. Show that they cannot reduce to couple. 4

ii) A particle of mass *m* is projected from a fixed point O into the air with velocity *u* in a direction making an angle α with the horizontal. Show that it describes a parabola. 6

d) i) Three perfectly elastic balls of masses m_1, m_2 and m_3 are placed in a straight line. The first impinges directly on the second with a velocity *u* and the second impinges the third. Show that the third ball will move after impact with velocity *u* if $(m_1 + m_2)(m_2 + m_3) = 4m_1m_3$. 5

ii) A bullet of mass *m* moving with velocity *u* strikes a block of mass *M*, which is free to move in the direction of motion of the bullet and is embedded in it. Show that the loss of K.E. is $\frac{1}{2}\left(\frac{mM}{m+M}\right)u^2$. 5

e) i) The velocities of a particle along and perpendicular to the radius vector from a fixed origin are λr and $\mu\theta$. Find the path and show that the acceleration, along and perpendicular to the radius vector are $\lambda^2 r - \frac{\mu^2\theta^2}{r}$ and $\mu\theta\left(\lambda + \frac{\mu}{r}\right)$. 6

ii) Find the C.G. of the plane lamina bounded by the parabola $y^2 = 4ax$ and its latus rectum lying in the first quadrant. 4
